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PLATO AND THE MATHEMATICIANS : AN EXAMINATION OF PROFESSOR HARE'S VIEWS[1]

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In his article " Plato and the Mathematicians " in *New Essays on Plato and Aristotle*, ed. Bambrough, Professor R. M. Hare offers an interpretation of the criticisms of mathematicians made at *Rep.* 510b ff. and of the contrast between mathematical and dialectical methods made in that passage and at 533b ff. It is plain from the text that Plato's criticism is twofold, first, that mathematicians use diagrams and secondly, that they start their inquiries from *hypotheses*, which they leave in some sense unexamined, instead of giving an account of them. Hare's main concern is to elucidate the sense in which mathematicians are said to fail to give an account of their *hypotheses*. On his view, to give an account of a *hypothesis* must be to give a definition of it. Hence the *hypotheses* in question cannot be any kind of proposition, since it makes no sense to ask for or to attempt to give a definition of a proposition. (Hare takes it for granted, no doubt rightly, that Plato saw that point.) Rather they must be things which can be defined, to be precise they are the actual entities which the mathematician studies, e.g. the odd, the even, the various geometrical figures and the three kinds of angle (of which Plato says at 510c that the mathematicians " make them their *hypotheses* "). Plato's charge, then, is that mathematicians introduce such entities into their discussions without defining them and proceed to derive concerning them conclusions whose truth cannot be guaranteed,

[1]I am grateful to Professor Hare for his comments on an earlier draft of this paper, and to various other colleagues, in particular Mr. J. H. W. Penney, Mr. J. C. B. Gosling and Mr. D. B. Robinson for their helpful criticism and suggestions.

since they follow not from propositions true by the definitions of the entities involved, but from propositions concerning those entities based only on intuition or empirical observation. An example of this faulty procedure is given by Socrates' geometrical "demonstration" in the *Meno* (82a ff.), where he begins by *drawing* a square (thereby introducing it into the discussion) and goes on to "prove" certain things about it, without at any stage giving a definition of what a square is. While, therefore, there is some evidence that definitions were given by the mathematicians of Plato's time, if we accept Hare's interpretation we may conclude that they did not realize the essential logical role of definition in proof, and were consequently casual about supplying and justifying definitions, sometimes omitting them altogether. "To suppose that this was so is at least plausible" (Hare, p. 28).

This very cautious phraseology is perhaps a trifle misleading. Apart from the *Meno* passage, whose evidential value I shall discuss briefly below (p. 200), Hare offers no evidence of what the practice of the mathematicians of Plato's time *was in fact*; the form of his argument is 'If my interpretation of *Rep.* 510b ff. is correct, this is what their practice *must have been*'. If, therefore, he offered that interpretation as merely plausible, it would be reasonable for him to suggest that his account of the mathematicians' practice is merely plausible also. In fact, he offers his interpretation as not merely plausible, but as necessary to account for what Plato says. Thus the *hypotheses* "must be things of which (or about which) it is possible to give a *logos*; and this, we now see, means 'to say what they are'" (p. 22). And again "the *hypotheses* here must be things, not propositions . . . it is impossible for them to be propositions here . . ." (p. 23). Hare, then, is committed to the view that because his interpretation of *Rep.* 510b ff. is certainly true, the account of the mathematicians' practice (or, more strictly, the account of Plato's view of the mathematicians' practice) which it entails is certainly true, not just plausible.

I shall argue that Hare's interpretation of the *Republic* passage, while not without plausibility when considered in isolation from the other contexts in which Plato uses the term *hypothesis*, appears insufficiently established when viewed in the light of those passages. Further, the *Meno* passage which Hare uses as an illustration is of no evidential value in this context. Thirdly, although the fragmentary state of the evidence for the mathematics of Plato's time and earlier makes it impossible to establish with certainty that mathematicians generally realized the essential role of definitions in proof, there is sufficient evidence to establish that the giving of definitions was an accepted part of mathematical procedure by the time of Plato. For these reasons Hare's interpretation is not to be preferred to the traditional view according to which the *hypotheses* of the mathematicians are some sort of propositions.

The first ground of objection to Hare's interpretation is that the sense of *hypothesis* which it requires, viz. 'postulated entity', appears to be unparalleled in Plato's writings. As is generally accepted by writers on

Plato, the verb *hypotithesthai* has the sense 'propose for discussion' or 'put forward as a basis for argument', with the implication that what is thus put forward is not established by argument or derived from anything but simply taken as given. Generally what is proposed is a proposition, frequently an existential proposition or its negation (e.g. *Phaedo* 100b 5-7, *Parm.* 135e 9-136a 2), but sometimes a proposition of another form (e.g. *Prot.* 339d 2-3 "he laid it down (*hypetheto*) that it is hard for a man to be truly good"). Sometimes, indeed, the grammatical object of the verb *hypotithesthai* designates, not a proposition, but some postulated entity (e.g. *Tim.* 48e 5-6, 53d 4-5). This is clearly the sense which Hare has in mind, but this use of the verb affords in itself no support for his suggestion about what Plato means by *hypotheses* in *Rep.* 510b ff., since the noun *hypothesis* appears in no other passage in the sense of 'postulated entity'. Apart from its uses in the senses of 'intention', i.e. what one *proposes* to do (*Gorg.* 454c 4 ; *Laws* V, 743c 5, VII, 812a 4-5), and 'plan', i.e. a method of procedure *assumed* at the outset (*Rep.* 550c 6) in every passage which I have come across the noun has the sense of "assumption' or 'supposition', i.e. some proposition assumed to be true ; in some cases the proposition is actually referred to (e.g. *Phaedo* 94b 1-2, *Meno* 87d 3), in others the propositional nature of the *hypothesis* is made clear from the context (e.g. *Theaet.* 183b 3-4, "on their own assumption (viz. that everything is in flux, a 3) they are not able to say anything" ; *Euth.* 11c, where Euthyphro's *hypotheses* won't remain stable, i.e. his assumptions contradict one another).[2] That this sense of *hypothesis* rather than that suggested by Hare is in question in the *Republic* passages also is suggested by the fact that the collection of Platonic definitions, which it is reasonable to accept as representing views current in the Academy, includes alternative definitions of *hypothesis* as 'undemonstrated first principle' and 'summary of a *logos*' (415b 10). Obviously, only the first sense is relevant to our present question,[3] and a *hypothesis* in that sense must be a proposition, since nothing but a proposition can be demonstrated or left undemonstrated. For the Academy, therefore, as for Aristotle (e.g. *An. Post.* I, 2 & 10, discussed below, pp. 198-9), a *hypothesis* as an element in philosophical or scientific reasoning was some sort of proposition.

It may, however, be urged in favour of Hare's interpretation that at *Rep.* 510c Plato describes the mathematicians as not only positing (ὑποθέμενοι) the odd, the even, etc., but also as making these very things their *hypotheses*, or treating them as *hypotheses* (ποιησάμενοι ὑποθέσεις αὐτά, c 6). Further, though the sense 'postulated entity' is certainly rare (v. *Liddell & Scott* s.v.), it does occur in what is apparently the earliest use of the word, the

[2]The other passages are : *Meno* 86e-87a, 89c ; *Phaedo* 92d, 101d, 107b ; *Parm.* 127d, 128d, 136a-b, 137b, 142b-c, 160b, 161b ; *Soph.* 244c.

[3]I do not know of any occurrence in the dialogues of *hypothesis* in the sense of 'summary' (e.g. the summaries prefixed to the plays in some MSS. of the Attic dramatists).

treatise *On Ancient Medicine*, generally dated to the last third of the fifth century.[4] The opening sentence of that work runs as follows :

ὁκόσοι μὲν ἐπεχείρησαν περὶ ἰητρικῆς λέγειν ἢ γράφειν ὑπόθεσιν αὐτοὶ αὑτοῖς ὑποθέμενοι τῷ λόγῳ θερμὸν ἢ ψυχρὸν ἢ ὑγρὸν ἢ ξηρὸν ἢ ἄλλο τι ὃ ἂν θέλωσιν, ἐς βραχὺ ἄγοντες τὴν ἀρχὴν τῆς αἰτίης τοῖσιν ἀνθρώποισι νούσων τε καὶ θανάτου καὶ πᾶσι τὴν αὐτὴν ἓν ἢ δύο ὑποθέμενοι, ἐν πολλοῖσι μὲν καὶ οἷσι λέγουσι καταφανέες εἰσὶν ἁμαρτάνοντες. . . .

Here a postulated entity, viz. the hot, the cold or whatever else medical theorists posit as the cause of disease, is unambiguously termed a *hypothesis*. Yet while this particular passage undoubtedly provides a parallel for Hare's suggested sense of *hypothesis*, the treatment of the term in the treatise as a whole is instructive for our discussion. In the same chapter as that from which comes the sentence just quoted, the author of the treatise contrasts medicine with speculative cosmology :

> I do not, therefore, think that it (i.e. medicine) requires any new *hypothesis*, as do obscure and problematical subjects, about which anyone who attempts to say anything is obliged to use a *hypothesis*, as in the study of the heavens or of things underground.

Here the author has apparently shifted from the sense of ' postulated entity ' to that of ' assumption ', since the most natural interpretation of the passage is that those who treat of such obscure matters have to make unverifiable assumptions. The transition is no doubt facilitated by the fact that the assumptions referred to are or at least include existential assumptions. Again, in c. 13 the author returns to the criticism of those who conduct their medical inquiries " from a *hypothesis* ". " If ", he says, " there is some hot or cold or dry or wet which is what makes a man ill, and if the correct treatment is to use the hot as a remedy against the cold, etc., then let us take a man of weak constitution, . . ." The conclusion is practically irresistible that the *hypothesis* in question is the proposition (or rather propositions) specified in the protasis of the conditional, *that* there are certain entities which cause disease and that the correct method of treatment is such and such. (' If ' has here the sense of ' assuming that '.) The treatise does not, then, provide an example of *hypothesis* in the sense of ' postulated entity ', where that sense is clearly distinguished from that which became normal in scientific contexts, viz. ' proposition assumed for the sake of argument '. And since, as we have seen, it is the latter sense which is universal in Plato outside the *Republic*, it is to be preferred there too unless the context clearly requires otherwise. In that case, 510c 6 should be regarded as an undifferentiated use similar to that in *On Ancient Medicine*.

This brings us to Hare's main argument in favour of his interpretation. If it shows that his interpretation explains something which the traditional interpretation leaves unexplained, then the former should be adopted ; but if the traditional interpretation will do as well, then it is to be preferred for the reason given above. Hare's contention is that *hypothesis* in 510b ff. must have the sense of ' postulated entity ', since there Plato finds fault with mathematicians for not giving an account of their *hypotheses*, while comparison with 533b ff. shows that by ' give an account ' Plato means

[4]On the date v. G. E. R. Lloyd, " Who is attacked in On Ancient Medicine ? ", *Phronesis* viii (1963), pp. 108-26, with the references to earlier literature given there.

'give a definition'. The strongest support for this view is 533b-c, where the distinguishing feature of dialectic is that it alone of all sciences "attempts systematically and in all cases to determine what each thing really is" (tr. Shorey (Loeb)). The reason why mathematics is unable to do this is that it leaves its *hypotheses* undisturbed, not being able to give an account of them. Here, then, we have a close link between saying what something really is and giving an account of *hypotheses*; if the traditional interpretation fails to explain this link, then Hare's view is to be preferred. To see whether the traditional interpretation is adequate in this respect we must first ask (*a*) what sense of *logon didonai* does it require? (*b*) what propositions does it regard as the *hypotheses* of mathematics?

The required sense of *logon didonai* is 'give a proof'. The use of *logos* in this sense is standard in Aristotle (Bonitz, *Index*, p. 435a 26-45),[5] while the occurrence of *logon didonai* in this sense in Plato is attested by, e.g., *Charm.* 165b 3-4 and *Phaedo* 95d 6-8 & 101d 1-e 3. The latter passage, which is crucial for our discussion, forms part of the well-known section, beginning at 100a, in which Socrates outlines his hypothetical method of scientific procedure. At 100a Socrates says that his method is to assume the *logos* which he judges strongest, and to take as true whatever "agrees" with it and false whatever does not agree. The much-debated question of the sense of 'agree' is not our concern; it is, however, clear that the assumption is a proposition and that the criterion of the truth and falsehood of other propositions is some logical relation which they have to that original proposition. Socrates immediately (b 4-7) specifies the propositions that he is assuming in the present argument; they are the propositions that Beauty, Goodness, Largeness and "all the others" exist. The later passage concerns the validation of an assumption; if one's assumption is challenged, one should ignore the challenge until one has tested the consequences of the assumption for "agreement" or "disagreement" (this time with each other, not, as earlier, with the original assumption). But if, after that procedure is complete, one is required to "give an account" of one's assumption, one should do this by "assuming whichever of the higher assumptions seems best, until one reaches something sufficient" (d 5-e 1). Since what has been assumed has been stated earlier to be a proposition, there can be no question of taking *logon didonai* here in the sense of 'give a definition'; rather it seems clear that Plato is talking of giving a proof of a previously unproved assumption by deriving it from some other assumption which is "higher" in the sense of logically prior.

It is easy to give an interpretation of the *Republic* passages on this deductive model. The mathematicians take as their starting-point certain unproved assumptions (cf. the Academic definition of *hypothesis* mentioned above (p. 195)). Plato refuses to accept these as given, and demands that the mathematicians "give an account" of them in the sense of deriving them from some more basic principle or principles. In order to see somewhat more clearly how Plato envisages this process we must now turn to

[5] v. also *Met.* Γ6, 1011a 8 ff., a particularly instructive passage.

our second question, viz. on the traditional assumption that the *hypotheses* of the *Republic* are propositions, what propositions are they ?

The only clue in the text is the reference at 510c 3-5 to the mathematicians' " assuming the odd and the even and the figures and the three kinds of angles and other things of that sort appropriate to each subject " ; an obvious suggestion is that the *hypotheses* are the propositions that these things exist. The construction with a direct object after the verb gives some support to this suggestion, since sometimes it appears that Plato regarded as equivalent the expressions ὑποτίθεσθαί τι and ὑποτίθεσθαί τι εἶναι .[6] But it would be unwise to put too much weight on that consideration alone, since the same construction seems at times to have a less specific sense. Thus at *Tim.* 53c-d the physical elements, fire, earth, etc., are said to be made out of the basic stuff of the universe in triangular shapes. It appears that Plato's words at d 4-6, ταύτην δὴ πυρὸς ἀρχὴν καὶ τῶν ἄλλων σωμάτων ὑποτιθέμεθα should be taken in some such sense as ' we assume this as the basic principle of fire and the other bodies ', i.e. what is assumed is not only the existence of something, but its functioning as a basic cause. Similarly, *Epin.* 977a-d gives an account of the fundamental place of the knowledge of number in all knowledge whatever, concluding with the words οὕτως ἀριθμὸν μὲν ἀνάγκη πᾶσα ὑποτίθεσθαι. Here again the direct object construction refers in an abbreviated form to propositions about the thing in question ; what we must assume is not that number exists, but that knowledge of number is that knowledge the lack of which would make man the most unintelligent of creatures (976d 5-8). Hence we must bear in mind the possibility that by ' assuming the odd and even, etc.' Plato means assuming not only that there are such things, but also some propositions about them, e.g. that every number is either odd or even.[7] In c. 2 of *An. Post.* I Aristotle points out that every science starts from a number of assumptions of different kinds, including definitions of the basic terms, axioms (i.e. propositions which must be accepted if anything is to be learned, presumably laws of logic) and propositions appropriate to the particular subject which must be accepted if that subject is to be learned. These latter Aristotle calls *hypotheses*, and seems to class existential propositions concerning the entities with which the subject deals as a sub-set of these.[8] As this classification has a certain

[6] *Parm.* 136a-c ; *Tim.* 61d. In the former passage the assumptions are introduced first as pairs of propositions (πολλά ἐστι, οὐκ ἔστι πολλά, etc.) in the interrogative form prefaced by εἰ. Then they are referred to by the phrase περὶ ὅτου ἂν ἀεὶ ὑποθῇ ὡς ὄντος, καὶ ὡς οὐκ ὄντος, and finally by the direct object construction, ἐάντε ὡς ὂν ὑποθῇ ὃ ὑπετίθεσο, ἄντε ὡς μὴ ὄν . In the latter passage the existential nature of the assumption is clear from the context.

[7] So Taylor, *Mind* xliii (1934), pp. 81-4.

[8] Most modern writers, including Ross in his summary of the chapter (*Aristotle's Prior and Posterior Analytics*, p. 508), Mure in the Oxford translation, Heath (*Mathematics in Aristotle*, p. 55), Lee (*CQ.* xxix (1935), pp. 113-8) and Cornford (*Mind* xli (1932), p. 41), confine the term *hypothesis* to existential assumptions, no doubt on the strength of Aristotle's examples. This view is shared by Pacius (*In An. Post. I*, c. 2, sect. 15) and Zabarella (*In An. Post. I*, c. 2, sect. 14). But (*a*) Aristotle says that a *hypothesis* asserts one of a pair of contradictories, which gives a wider sense than ' existential assumption ' ; (b) the wider sense is nearer that given to the word in c. 10, where

degree of correspondence with Euclid's division of his basic elements into definitions, " common notions " of general application and postulates appropriate to his particular subject matter,[9] it is likely that some classification of this kind was generally recognized. Since there is no hint in Plato's discussion of any differentiation between kinds of basic assumption it seems reasonable to suppose that he is concerned with all the kinds without distinction. This raises the question whether Plato includes the definitions of mathematics themselves under *hypotheses*. At *An. Post.* I, 2, 72a 18-21 Aristotle distinguishes definitions from *hypotheses* on the ground that a definition asserts neither that anything is the case nor that anything is not the case, whereas a *hypothesis* asserts either that something is the case or that something is not the case.[10] That distinction is neither explicitly stated nor implied, to my knowledge, anywhere in Plato, while in three passages (*Euth.* 9d 1-8 ; *Charm.* 163a 6-7, 172c 8-9) we find *hypotithesthai* used of a definition. Plato, then, would regard it as sometimes correct to describe someone who gives a definition as *hypotithemenos ti*, and in default of any evidence to the contrary it is safest to assume that in the *Republic* he includes definitions among the *hypotheses* of mathematics.

On this view the mathematicians are being accused, not of failing to give definitions of things which they ought to define, but of taking for granted certain propositions, including definitions, which they ought to prove. It is not enough for a mathematician to *say* " There are numbers " or " A triangle is a plane figure enclosed by three straight lines "; he must *prove* that there are numbers, or that that is what a triangle really is. Further, 511b-d, where Plato says that there is an unhypothetical first principle of everything from which dialectical reasoning proceeds, and that the subject-matter of mathematics is intelligible with a first principle, strongly suggests that the deficiency is to be made good by deriving the basic propositions of mathematics from some self-evident principle common to the whole of knowledge. It is, moreover, plain from the immediately preceding simile of the sun that this basic principle is some proposition concerning goodness, perhaps the definition of goodness. Thus at 509b 6-9 goodness is said to confer on things that can be known not only their capacity to be known but also their being and their nature ; I take this to mean that goodness not only renders intelligible objects intelligible, but also accounts for their

it means any provable assumption accepted without proof, e.g. that a certain line is straight ; (*c*) the wider sense is assumed at *Phys.* 253b 2-6, where the *archai* of physics include the *hypothesis* that nature is a principle of change. That sense is understood by Themistius (*C.A.G.* V, i (Wallies) p. 7) and Philoponus (*ibid.* XIII (Wallies), pp. 34-5), whose examples include Euclid's first and third postulates and the maxim ' Nothing can come into being out of nothing ', and by Aquinas (*In An. Post. I*, lect. v), whose example is Euclid's fourth postulate ' All right angles are equal '.

[9] v. preceding note, and Lee *op. cit.* and Heath *op. cit.*, pp. 53-7.

[10] I, 10, 76b 35-6 is generally taken in the same sense, but for a persuasive alternative interpretation v. von Fritz, *Archiv für Begriffsgeschichte* I (1955), pp. 38 ff. I am not, however, persuaded by his main contention that *An. Post.* I, 10 is directed expressly against Plato's account of the fundamentals of mathematics at *Rep.* 510b ff., since the connection between Aristotle's examples and Plato's appears to me more tenuous than von Fritz allows.

existing at all and for their being what they are. And since the objects of mathematics, whether Forms or not, are certainly among the things which can be known, we may take Plato to be saying that we can be sure that they really exist, and that their properties are as we say they are, only when we see how their existence and their having just these properties is necessitated by the nature of goodness. That is to say, we can have that assurance only when we can derive the propositions asserting the existence and defining the nature of these entities from some proposition which exhibits the nature of goodness.

It seems to me that this gives a perfectly clear and intelligible account of the connection made at 533b ff. between grasping completely and systematically what things are and getting rid of *hypotheses*. Dialectic grasps the nature of things because it alone can give assurance, as opposed to mere supposition, that things exist and that they are as we say they are. Mathematics, on the other hand, while indeed it discovers the truth about its objects, and so " grasps reality to some extent " (b 7), cannot give that assurance so long as it begins from unexamined *hypotheses*, and so remains in a state of dreaming of things which it can't see in waking life (b 8-c 3). The traditional view, then, accounts adequately for the passage which gives the strongest support to Hare's interpretation.

As regards the examination of the *Republic* passages alone, therefore, Hare has shown insufficient ground for adopting his interpretation in favour of the traditional view. This leaves *Meno* 82a ff. to be considered. On this I can say only that it seems to me to be no evidence at all for the practice of mathematicians of Plato's day, for two very obvious reasons, firstly that Socrates was not a professional mathematician, and secondly that the passage is not a piece of formal mathematical work, but an informal bit of instruction conducted not for any mathematical purpose but merely to establish the metaphysical thesis that learning is recollection. Again, the fact that *in a particular demonstration* a mathematician does not define his terms is not sufficient for Hare's interpretation of Plato's critique. A geometer, for instance, is surely not required to put the definition of e.g. ' line ' at the head of every theorem concerning lines, but is entitled to presuppose it, having once stated it at the beginning of the whole system. So if Hare is to represent Plato as saying something not thoroughly and patently unreasonable, he must take Plato to say that it was a *characteristic* feature of mathematical practice to leave the objects of the study *altogether* undefined. The word ' characteristic ' is worth stressing ; since Plato is saying that what distinguishes mathematics from dialectic is that the former uses diagrams and starts from *hypotheses*, whereas the latter does not use diagrams and eliminates all *hypotheses*, he must be understood as saying that the practice of mathematicians *as a rule* exhibits those features, not just that some careless mathematicians sometimes talk or write in that way. Some careless philosophers sometimes produce invalid arguments, but one could not reasonably argue on that ground that what distinguishes philosophy from, e.g., science is the fact that philosophical arguments are invalid whereas scientific ones are valid. The *Meno* passage seems to me to give no support to any generalization about the practice of mathematicians of the time of Plato, and *a fortiori* not to support the generalization which Hare requires.

It remains only to add that the ancient evidence suggests that the credit for having first realized the essential logical role of definitions in proof belongs not to Plato but to the Pythagoreans. Aristotle says (*Met.* A5, 987a 19-21) that they were the first to speak about the essence of objects and to define them, adding that their definitions were superficial. Two passages of later writers point to the same conclusion : Diogenes Laertius (VIII, 48) cites Favorinus as saying that Pythagoras used definitions throughout his mathematical subject-matter, and that he was followed in this by Socrates and his associates (obviously including Plato), while in Proclus' summary of the history of geometry (*In Eucl. I*, ed. Friedlein, pp. 64. 16—70. 18) we read that " Pythagoras transformed this study into the form of a liberal education, examining its principles from the beginning and tracking down the theorems immaterially and intellectually " (tr. Thomas, *Greek Mathematical Works* (Loeb) v. I, p. 149). It seems clear that Proclus regards Pythagoras as having put the subject on a scientific footing by, among other means, systematically setting out its basic principles, which must have included definitions. Examples of Pythagorean definitions are given by Nicomachus (*Intr. Arithm.* I, 7, 3-4), Iamblichus (*In Nicom. Intr. Arithm.* 10-12) and Porphyry (*In Ptol. Harm.*, ed. Düring, pp. 92-3, 107-8 ; *D.-K.* 47 A 17, B 2 (Archytas)), while Heath (*The Thirteen Books of Euclid's Elements*, v. II, pp. 294-5) accepts that the substance of Bks. vii-ix (including the definitions prefaced to the seventh book) goes back at least to the Pythagoreans, and that there is " a clear indication of the existence at least as early as the date of Archytas (about 430-365 B.C.) of an *Elements of Arithmetic* in the form which we call Euclidean ". But while it appears that the Pythagoreans introduced definition into mathematical reasoning, or were at least the first to use definitions systematically, the use of definitions was not confined to them. In another passage of the *Metaphysics* (Z11, 1036b 8-13) Aristotle says that the Pythagoreans object to the definition of the triangle and the circle in terms of line and continuity, which indicates that there were non-Pythagorean definitions of geometrical figures. Again, Proclus in his summary says that Hippocrates of Chios[11] was the first to write " elements ", which in the context we are justified in taking to mean that he was the first to compose a systematic treatise on the lines of Euclid ; such a treatise must have included definitions. Finally, we have a piece of evidence from Plato himself ; in the one passage of his works known to me where a practising mathematician is represented as describing mathematical method (*Theaet.* 147c 7-148b 2), Theaetetus begins his account of his demonstration of the incommensurability of square roots by defining his technical

[11]Active in the late 5th cent. Proclus says that he preceded Plato, and implies that he was contemporary with Theodorus, who was a friend of Protagoras (*Theaet.* 162a).

terms ' square ' and ' oblong ' number, ' quantity ' and ' power '. It appears, therefore, that by the time of Plato the practice of giving definitions at the outset of mathematical treatises, introduced by the Pythagoreans, had become part of the regular method of mathematicians of all schools, and that there existed compendious works on geometry and arithmetic which displayed this feature.

I conclude, therefore, that Hare's interpretation does not offer a better understanding of the Republic passages than does the traditional view, while the latter is to be preferred on the grounds (*a*) that it takes *hypothesis* in the sense which prevails in the writings of Plato and his immediate successors, and (*b*) that, unlike Hare's view, it is consistent with what little evidence we possess about the role of definition in the mathematics of the time.

An interesting feature of the version of the traditional view which I have defended is that it applies to mathematics a principle of cardinal importance in Plato's thought, viz. that something is fully explained only when it is shown that it is best that it should be as it is. For to say (as on p. 200 above) that the nature of goodness necessitates that mathematical objects should be of such and such a nature is in effect to say that they are so because it is best that they should be so. Socrates explains in the *Phaedo* (97c ff.) how his insistence on the necessity of this kind of explanation led him to reject the cosmological speculation of his day. There and in the *Timaeus* this teleological principle serves as a pattern of explanation of events in the physical world ; in the *Republic* it serves to explain states of affairs which we are accustomed to regard as obtaining necessarily. This does not, however, represent any departure from the view expressed in the *Phaedo*, since Socrates there makes clear that the questions that led him to formulate his principle were not only questions about the natural world, but also mathematical questions (96e-97b), while there is no suggestion that one kind of question requires a different kind of answer from the other. Rather, since the second-best explanation (i.e. explanation in terms of Forms) applies to all states of affairs without exception, it is likely that the best sort of explanation (i.e. teleological explanation) was also thought to be of quite unrestricted application.

An interesting conclusion follows from this, viz. that though Plato certainly thought knowledge of necessary truths the best kind of knowledge, and perhaps the only kind of cognizance which deserves the name ' knowledge ', he nevertheless regarded it in a way more appropriate to the apprehension of contingent facts. For in order to understand the explanation of some state of affairs as obtaining because it is best that it should obtain, we must be able to envisage the possibility that some other, worse state of affairs should obtain instead. But to say that a state of affairs is necessary is to say that no alternative to it is possible ; there can, then, be no sense in the suggestion that a state of affairs obtains necessarily because it is best that it should. This may best be seen from an example ; Plato appears to think that it is true that all the radii of a circle are equal because that is best, i.e. because, while there might be circles with unequal radii, that would not be such a good state for things to be in. But there could not be such circles, since any figure with unequal radii would not be a circle. This particular example was deliberately chosen to show the parallel between Plato's view and that of Descartes, who writes[12] "God was just as much free to make it untrue that all straight lines drawn from centre to circumference are equal, as he was not to create the world". The major difference between their views is that for Descartes the choice of which set of necessary truths actually obtains is made quite arbitrarily by Gods choice, independently of any consideration of what is best, while for Plato it is determined by what is absolutely best, independently of the divine or any other will (indeed, in the *Timaeus* the Demiurge operates according to predetermined principles of what is the best arrangement of the universe). But both share the view that the necessary truths which in fact obtain are one among a number of alternatives. Both may therefore be seen as ancestors of modern conventionalist theories of necessary truth, Descartes holding that the convention which is adopted is arbitrary indeed but of divine origin, while Plato holds that the necessary truths that we accept are true because it is best that they should be. While I do not suggest that Plato in fact held that necessary truths in any way depend on linguistic convention, it is easy to see how his view could be transformed into the view that necessary truths are true because of linguistic conventions, and that one particular set of conventions has been adopted because it is the best. I do not wish to deny that Plato regarded the discovery of necessary (including mathematical) truth as the discovery of facts about a special realm of reality, or that the traditional view of a fundamental opposition between Platonism and conventionalism in mathematics is broadly correct; I wish merely to remark on an aspect of Plato's view of mathematical truth which lends itself readily to transformation into a conventionalist view.

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[12]To Mersenne, *Philosophical Writings*, ed. Anscombe & Geach, p. 262.